Inverse Problems in Optics

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INVERSE PROBLEMS IN OPTICS.

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Introduction.

The direct or "normal" problem in optical physics is to predict the emission or propagation of radiation on the basis of a known constitution of sources or scatterers. The inverse or indirect problem is to deduce features of sources or scatterers from the detection of radiation. An intuitive solution of the optical inverse problem is commonplace, we infer size, shape, etc. from their scattering and absorption as detected by our eyes. However, intuition has to give way to mathematical reconstruction if we wish to analyze optical data beyond their visual appearance. Examples are listed below.

FEATURES OF INVERSE OPTICAL PROBLEMS.

A general definition or inverse optical problems can be attempted as follows: We describe sources or scatterers by a set of space-time functions, the source functions, which are mapped by a set of (not necessarily linear) operators R into a set of functions D, which are the measured data: $R\{S\}=D$. For example, S can be a current distribution, and R the associated integral transformations, leading to the far zone radiation pattern, or electromagnetic field vectors, etc. The inverse problem is, ideally, solved by establishing an inverse operator R^{-1} such that $S=R^{-1}\{D\}$, which is consistent with the a priori knowledge i.e. the physical information coming from general hypothesis, principles, the constraints of the experimental set up, etc.

It is well known that the above inversion is connected with the mathematical questions of existence, uniqueness, and stability. For the physist is the problem of the existence of a solution less improtant than the problems of uniqueness and stability, as he already supposes that his data are generated by sources and scatterers, and that the operator R is known as well. However, the uniqueness is by no means certain since e.g. the mathematical construction of non radiating sources is possible. The stability of the inversion procedure may also be very poor, if one considers, for instance, object restoration beyond the diffraction limit: small errors in the data lead to large errors in the solution unless suitable stabilizing constraints are imposed.

Some of the above problems have been considered already a long time ago. Properties of non radiating sources were investigated about 70 years ago by Sommerfeld, Ehrenfest, and Herglotz. (see Hoenders [3]). Another longstanding inverse problem is the construction of diffuse

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reflection surfaces yielding a prescribed angular distribution of the average scattered intensity like Lambert's cosine distribution.

The sought features of the source or the scatterer are inferred from the measured data by virtue of the appropriate inversion relation, and accounting for the available prior knowledge. Detectors provide intensity data which includes also the modulus of the degree of coherence, and the intensity autocorrelation. We thus have to deduce the necessary phase information from the intensity distribution, which is known as the phase retrieval problem. Further steps in the reconstruction of scatterers are the reconstruction of the field up to the surface of the scatterer (inverse diffraction problem) and, finally, the reconstruction of the object from the field outside the scatterer (inverse scattering problem) along with the pertinent uniqueness and stability problems.

LIST OF SPECIFIC PROBLEMS.

1) Inverse Radiative Transfer: Remote sensing of flames, concentration profiles, or wind velocities. The inversion of the radiative transport equation leads to ill-posed problems.
2) Phase Retrieval Problems: The reconstruction of a phase from the knowledge of the modulus of a wave function. Only the modulus of the complex amplitude can be determined from measurement whereas the phase is indispensable for the determination of the structure of the scatterer. The phase retrieval problem arises in light- and particular in electron microscopy, because holography in electronmicroscopy is still a very difficult procedure. For a recent review see Ferwerda [2]. Note that holography is a procedure from which the phase uniquely can be retrieved in a simple way.
3) Inverse Diffraction: Determination of the field amplitude (or correlation) on a surface from the knowledge of the field (or correlation) on a surface to which it has propagated. Connected with this problem is the stability of the inversion, which leads to the concept of numbers of degrees of freedom of a wave field. The sinc type spatial correlation of the black body is obtained from Lambert's cosine distribution, Baltes [1].
4) Inverse scattering: Determination of the source distribution or scatterer from the complex amplitude of the scalar- or vectorial wave field behind the source or scatterer.
5) Uniqueness stability: The existence of "non radiating sources" prevents the unique determination of radiating current distributions, and only the use of a prior information can improve the situation. Scatterers to are in general not uniquely determined by the scattered wave function but suitable processing of independent measurements like the projections of an object can lead to a unique three dimensional determination of the object (tomography).
6) Reconstruction of the shape of a cavity from the eigenvalue spectrum: Can one hear the shape of a drum, can one see the shape of a cathedral or a black body. It can be shown that the shape of a drum is uniquely determined by its eigenvalue spectrum.

7) Retrieval of statistical features of random systems: Examples are rough surfaces, random distributions of random media.

RECENT PROGRESS.

Non radiating stochastic current correlations have been studied recently, and the condition for non radiating first order current correlations is shown to be

$\langle \tilde{\underline{j}}^T(k\underline{s}_1)\tilde{\underline{j}}^{T*}(k\underline{s}_2)\rangle=0$, where $\tilde{\underline{j}}^T(k\underline{s})=\underline{s}\times\underline{s}\times\tilde{\underline{j}}(k\underline{s})$, for all unit vectors $\underline{s}_1$ and $\underline{s}_2$, and $\underline{j}(k\underline{s})$ denotes the Fouriertransform of the current $\underline{j}$.

We will concentrate on the experimentally found large class of random scatterers who lead to K_ν correlated fields:

$\langle u(\underline{r}_1)u^*(\underline{r}_2)\sim \rho\ell^{-1}K_\nu(\rho\ell^{-1})$, where u denotes the complex amplitude, K_ν the modified Besselfunction of order ν, ℓ some coherence length, and $\rho=|\underline{r}_1-\underline{r}_2|$. These correlations can be understood from a micro area model with a suitable slope distribution.

1. Baltes, H.P., editor, Inverse Problem in Optics, Springer, 1978.
2. Ferwerda, H.A., Ch.2 of ref. 1 pp. 13-39.
3. Hoenders, B.J., Ch.3 of ref. 1 pp. 41-82.